

ASTM International Paper # 11569**Symposium on Probabilistic aspects of Life predictions****Sponsored by ASTM Committee E08 on Fatigue and Fracture****November 6-7, 2002****Fontain Hilton Resort Miami Beach, Florida, USA****PROBABILISTIC FRACTURE TOUGHNESS, FATIGUE CRACK GROWTH
ESTIMATION RESULTING FROM MATERIAL UNCERTAINTIES**

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A new approach is introduced in this paper that can evaluate fracture toughness of aircraft and aerospace alloys by using static parameters that are obtainable from full stress-strain curves available in the MIL-HDBK. With this approach, the energy absorption rates related to the plastic deformation at the crack tip and near crack tip are estimated and used to extend the Griffith theory of brittle fracture to fracture mechanics of ductile metals. An equation has been established that can define the critical crack length as a function of fracture stress. Having fracture stress and critical crack length on hand, the plane strain and plane stress fracture toughness can be calculated by applying the stress intensity factor equation. The calculated fracture toughness for 2219-T8, 2014-T6 aluminums, and Ti-6Al-4V Titanium alloys were compared against the experimental test data generated from reliable sources. Excellent agreement between test data and the theory were found. When fracture toughness was calculated by this method, fatigue crack growth curves for the above-mentioned alloys were then generated and compared with test data. The threshold stress intensity factor value, ΔK_{th} , in the region I of the da/dn curve, was approximated by establishing a point on the Kitagawa diagram associated with the region of linear elastic fracture mechanics. Results of ΔK_{th} value estimated by this method were in fine agreement with values observed for many materials in the NASGRO database. Probabilistic evaluation of fracture toughness, and fatigue crack growth analysis considered 5 to 10% coefficient of variation of material K_c , and K_{th} random variables. The probabilistic evaluation of the above selected alloys determined: 1) shift in both fracture toughness versus material thickness and fatigue crack growth (da/dn versus ΔK) plots for the above selected alloys, 2) sensitivity of the random variables (K_c , K_{th}) to response variables, 3) probability density function, and 4) cumulative distribution function demonstrating the probability of crack growth rate.

1.0 Introduction

In designing fracture critical components of aircraft or space structures, considerable attention must be given to the fracture toughness parameter and material ability to resist cracking during its service usage. The present approach to the life assessment of high strength or low fracture toughness materials is to use linear elastic fracture mechanics, which for the crack geometry in consideration utilizes the stress intensity factor, K , as the crack tip parameter. The critical value of K and fatigue crack growth rate properties (da/dn versus ΔK) must be available through the ASTM standards when assessing structural life in a load varying environment. Static and fracture characterization of a candidate alloy are usually costly and time consuming. In many cases numerous tests must be conducted in order to understand fracture properties variations with respect to orientations and temperature environments when estimating number of cycles to failure. An alternative approach is necessary in order to reduce the cost and time of testing, so that designers and analysts could perform structural sizing within the allocated budget and schedule. The remedy to this problem is to use the virtual testing approach, which utilizes the extended Griffith theory to calculate the amount of energy consumed at the crack tip for the presence of plastic deformation without using the ASTM standards.

The general principle from which the Griffith Theory was derived for elastic crack propagation is not limited to ideally brittle materials, such as glass. This theory applies as well when dissipative mechanisms such as plastic deformation, are present. The Griffith energy balance principle can be extended to apply to ductile metals where the crack tip exhibits considerable plastic deformation. The proposed virtual testing method will utilize the energy per unit volume under the full stress-strain curve and can determine the energy released rates for plastic deformation at the crack tip and near crack tip. Material variations and its effect on fracture properties (fracture toughness and fatigue crack growth) can be estimated through the probabilistic approach also described in this paper.

2.0 Technical Approach

2.1 Approach to Fracture Toughness Determination

Material residual strength capability curve (a plot of fracture stress versus half a crack length) can be generated through the extended Griffith theory [1,2]. Energy absorption rate for plastic deformation at the crack tip is calculated and used to establish a relationship between fracture stress and half critical crack length. The total energy per unit thickness absorbed in plastic straining of the material around the crack tip, U_P , can be written as:

$$U_P = U_F + U_U \quad 1$$

where U_F and U_U are the energy absorbed per unit thickness in plastic straining of the material beyond the ultimate at the crack tip and below the ultimate stress near the crack tip, respectively.

The extended energy balance equation, in terms of U_F and U_U , described by equation 1, can be rewritten as:

$$\partial[U_E - U_S - U_F - U_U]/\partial c = 0 \quad 2$$

where U_E and U_S are the total available energy and energy necessary to create two new crack surfaces. $g_1 = \frac{\partial U_E}{\partial c}$ and $g_2 = \frac{\partial U_U}{\partial c}$ are the rates at which energy is absorbed in plastic straining beyond the ultimate stress at the crack tip and below the ultimate stress near the crack tip, respectively. The extended Griffith theory in terms of g_1 and g_2 can be rewritten as:

$$\frac{\pi \sigma^2 c}{E} = 2T + \frac{\partial U_F}{\partial c} + \frac{\partial U_U}{\partial c} \quad 3$$

where $\partial U_S/\partial c = 2T$, the work done in creating two new crack surfaces. The derivation of the two terms g_1 and g_2 are available in References [1,2]. Having fracture stress and half critical crack length on hand, material fracture toughness can be calculated. Fracture Toughness Determination (FTD) software is available to estimate material fracture toughness. This code is able to generate the plane strain and stress fracture toughness and plots the variation of fracture toughness, K_c , versus plate thickness, t . Figure 1 illustrates the extended Griffith theory and regions of crack tip straining.

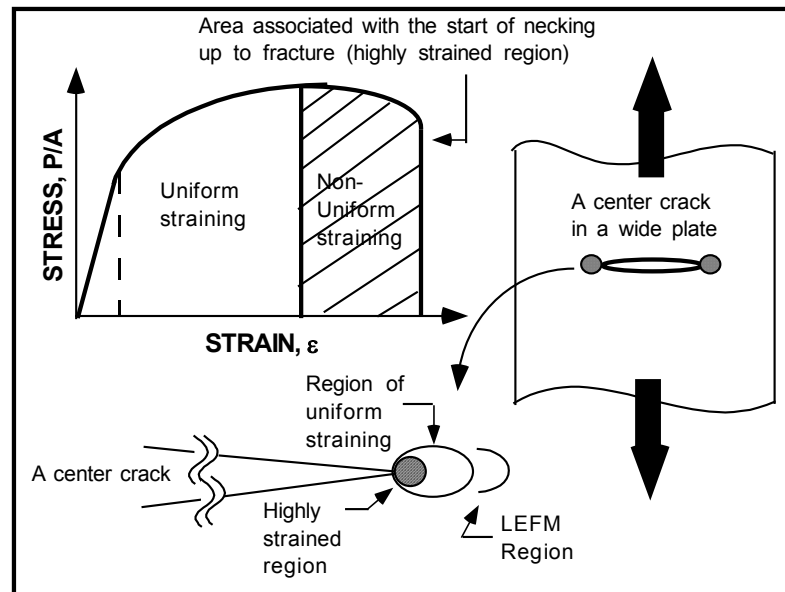


Figure 1: full stress-strain curve and crack tip deformation

2.2 Material Fatigue Crack Growth Rate Curve Without Using the ASTM Standards

Material fatigue crack growth rate curve can be generated by applying fracture toughness value, K_c , obtained from the extended Griffith theory, for region III of da/dn curve and the threshold stress intensity factor, K_{th} , (region I of da/dn curve) from Kitagawa diagram concept [3]. Additional two points were estimated in region II that enable to establish the Paris region of the da/dn curve. One of these points was taken just prior to the region III of the da/dn curve, where crack growth rate accelerates and the second one prior to the threshold region, where crack growth rate is decelerating, Figure 2 [3]. The

da/dn equation describing fatigue crack growth rate is similar to Forman and Newman equation used in NASGRO [4]. The constants used in the NASGRO to relate the crack growth rate to the stress intensity factor range were determined in NASGRO through physical testing. However, with the Fatigue Crack Growth (FCG) computer code, they can be obtained by the two following assumptions: 1) the crack growth rate of $\alpha_1 E^{-8}$ in./cycle, the corresponding $\Delta K = \alpha_2 \Delta K_{th}$, and 2) for the crack growth rate of $\beta_1 E^{-2}$ in./cycle the corresponding $\Delta K = \beta_1 K_c$, where α and β have been established based on experimental test data, which have a constant value for many aluminum alloys [5]. These two assumptions are sufficient to establish material crack growth rate curve (da/dn versus ΔK) with good accuracy. Two example problems that are used here to demonstrate the FCG capability in generating the fatigue crack growth rate curve for 2014-T6 and 2219-T87 aluminum alloys, respectively. The FCG software is available to generate material fatigue crack growth rate curve and assess the effect of material variation on the da/dn curve.

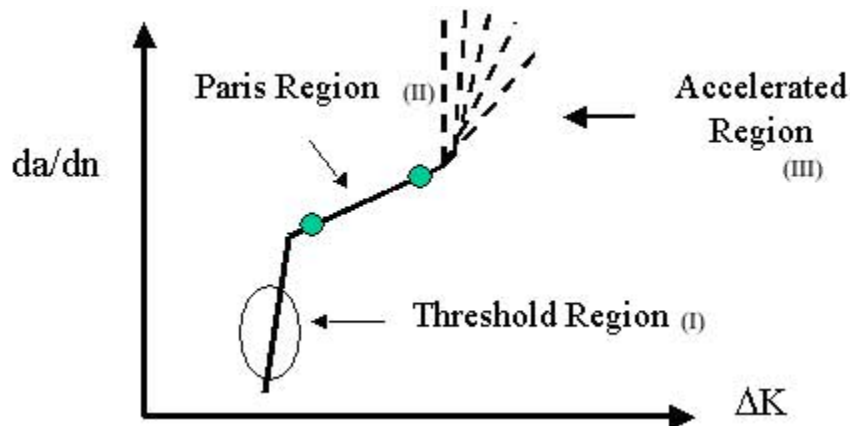


Figure 2: Regions of fatigue crack growth curve

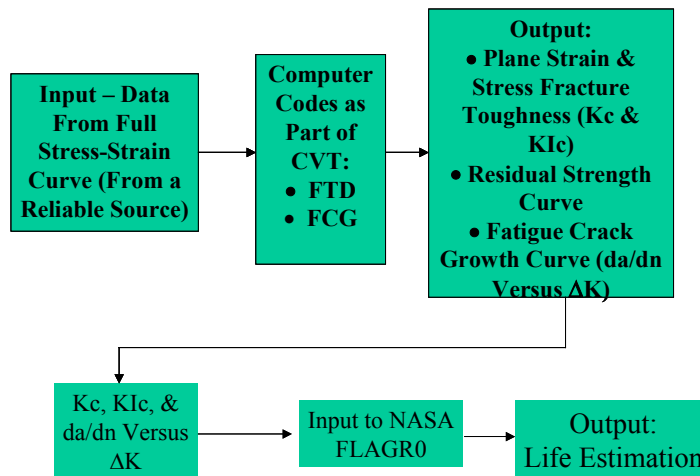


Figure 3: Input and Output from FTD and FCG

2.3 Application of FTD and FCG For Life Estimation

Both fracture toughness and fatigue crack growth rate data are needed to calculate number of cycles to failure. To calculate material fracture toughness for a given part thickness, certain data from full stress-strain curve are needed as an input to the FTD

software. Figure 3 illustrates input and output to the FTD and FCG software. Figures 4 and 5 show typical stress-strain curves for 2219-T87 and 2014-T62 aluminum alloys. Material fracture toughness as a function of part thickness can be calculated with the FTD software and are plotted in Figures 6 and 7 for 2219-T8 and 2014-T6 aluminum respectively. Two curves are plotted, which represent the the effect of plate width on fracture toughness (narrow and wide plates). For comparison with NASGRO data base [4], fracture toughness values for several part thickness are also plotted. Note that fracture

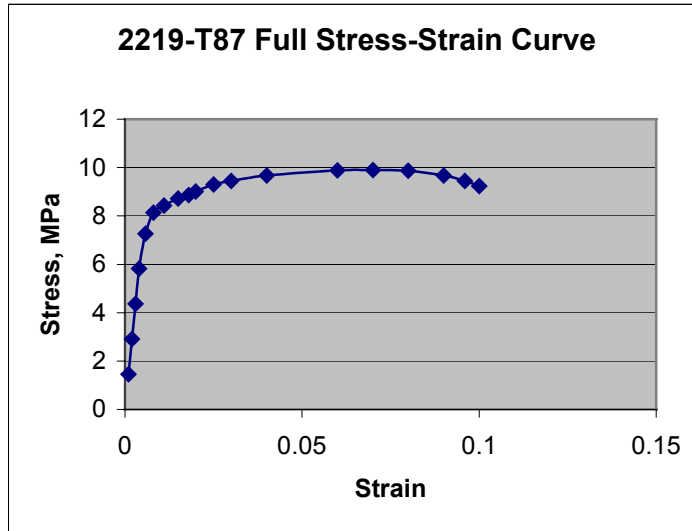


Figure 4: Full Stress-Strain Curve for 2219- T87

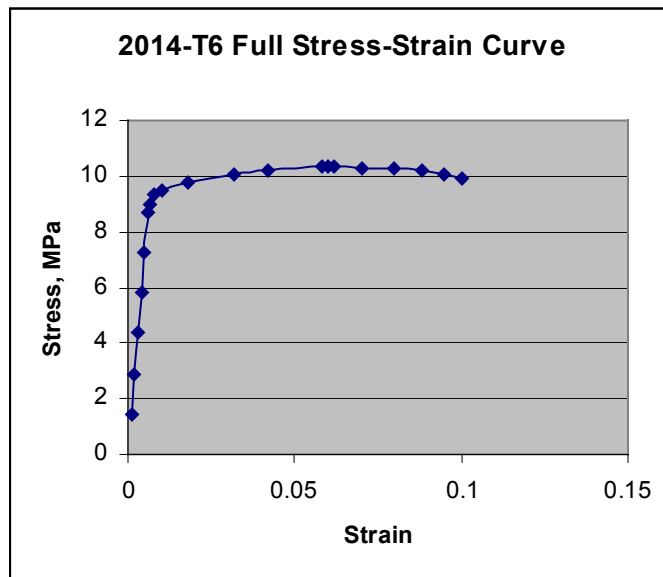


Figure 5: Full Stress-Strain Curve for 2014- T62

toughness and fatigue crack growth data in the NASGRO material library are available as average values. Excellent agreement between the physical testing data (NASGRO) and FTD computer code can be seen.

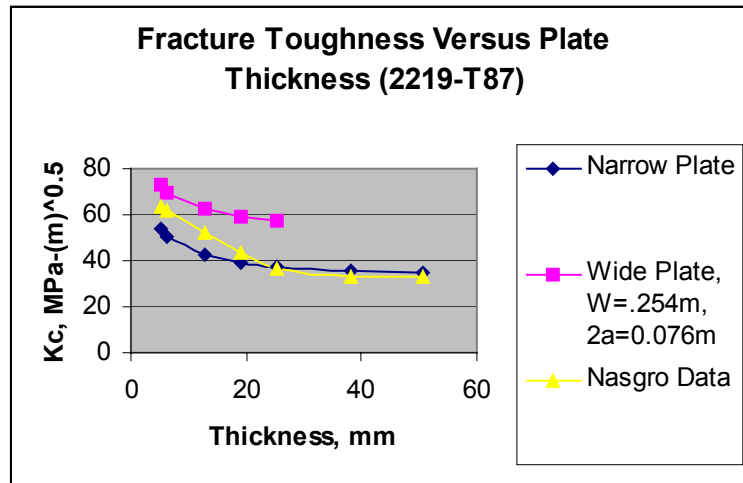


Figure 6: Material fracture toughness versus thickness for 2219-T87 (NASGRO data compared with virtual testing)

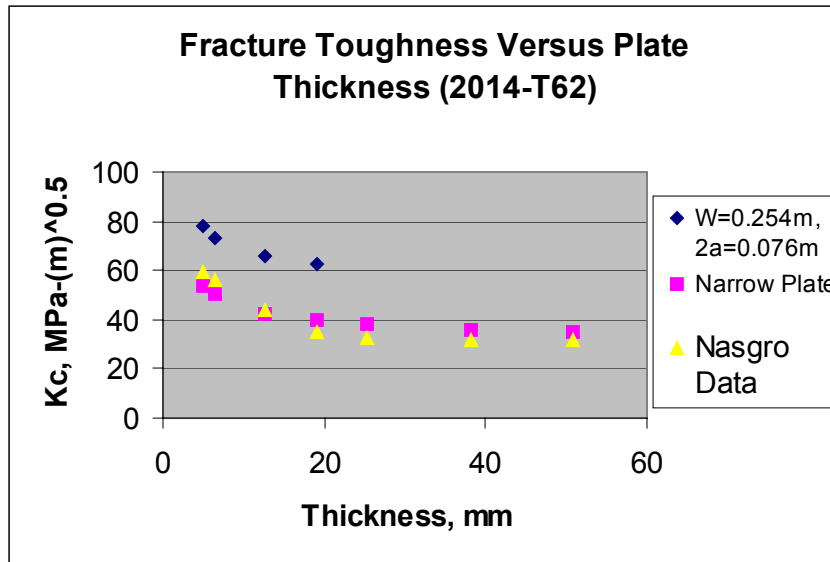


Figure 7: Material fracture toughness versus thickness for 2014-T62 (NASGRO data compared with virtual testing)

By estimating fracture toughness through the FTD, material fatigue crack growth curve can be plotted. The threshold value associated with region I can be estimated through Kitagawa diagram described in Reference [3]. The two points in the Paris region were calculated as having $K_c/K = 1.125$ and $K/K_{th} = 1.25$ values for crack growth rate of 0.005 and $1.0E-8$ in./in., respectively. Figures 8 and 9 are the da/dn versus ΔK for 2219-T87 and 2014-T62 aluminum alloys from NASGRO data base, which are compared with the virtual testing approach generated through FCG computer code.

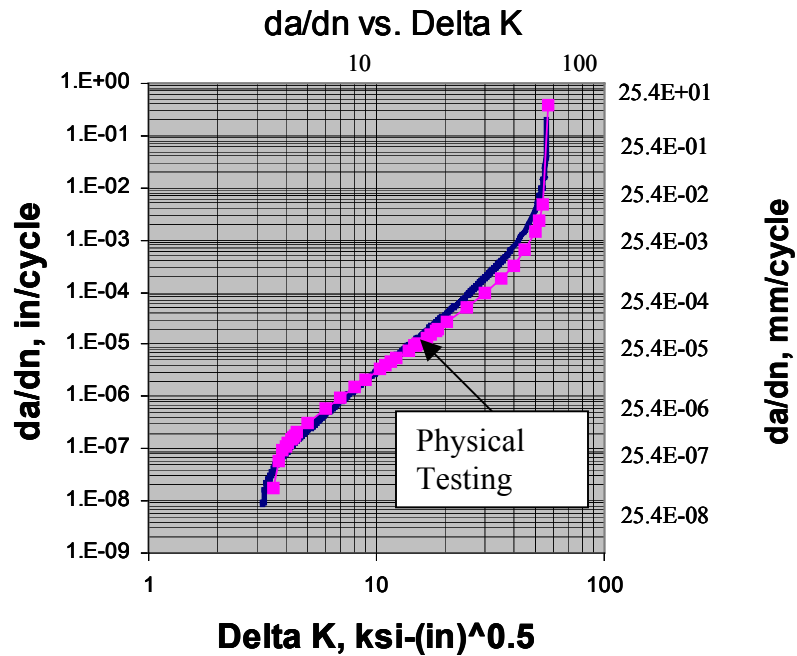


Figure 8 – Physical testing (NASGRO) versus FCG computer code (2219-T87)

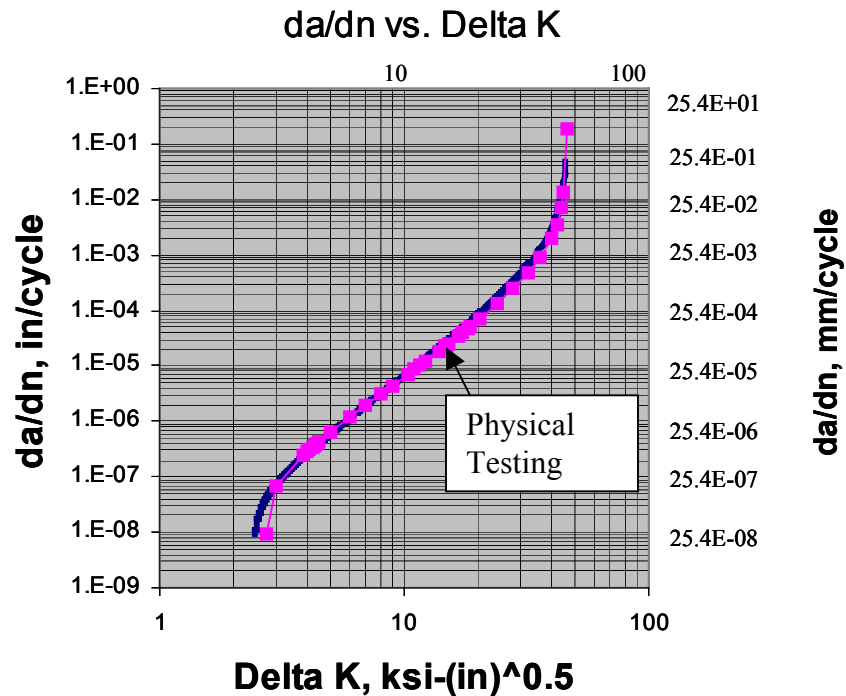


Figure 9 – Physical testing (NASGRO) versus FCG computer code (2014-T6)

2.4. Material Variation and Probabilistic Assessment for FTD and FCG

In obtaining material properties through physical or virtual testing, it is always expected to observe some amount of scatter on fracture toughness and fatigue crack growth values due to material variations that can vary through heat lots when the material is processed. This type of variation can be observed also in test coupons that have been machined from

a given plate of a given manufacturer by a specified heat lot. In the case of virtual testing approach proposed in this paper, material variation observed on a stress-strain curve has scatter results on the fatigue crack growth data, Figure 10.

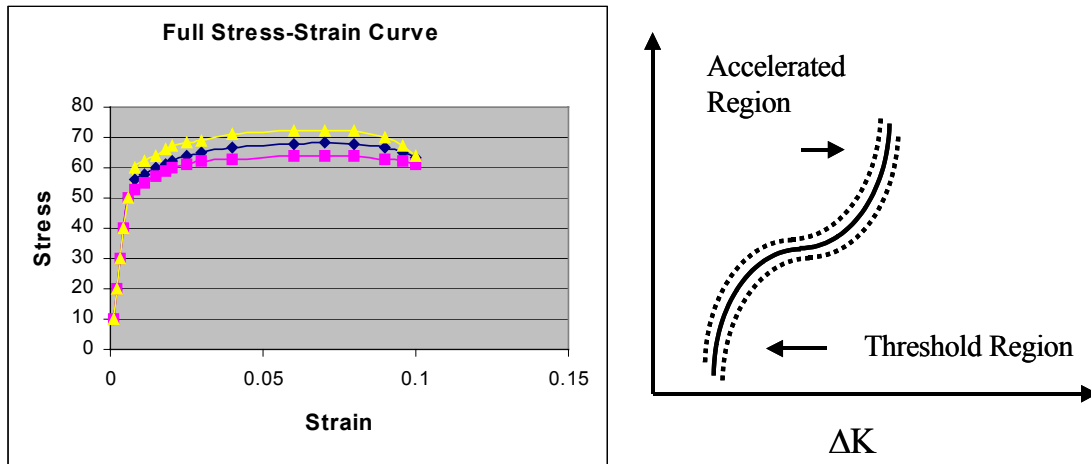


Figure 10: Material variation effect on fatigue crack growth data

2.5 Probabilistic Evaluation of Fracture and Fatigue Crack Growth

Alpha Star/NASA has developed GENPAM, a computer software for probabilistic material, and structural analysis that integrates probabilistic methods, with fracture mechanics. The software has been integrated as part of the GENOA suite of codes. Available probabilistic methods include: 1) Monte Carlo Simulation (MCS), 2) advanced reliability algorithms and 3) importance sampling methods. MCS, traditionally used for reliability assessment, is deemed computationally too expensive for large structures or structures with complex behaviors. Extensive effort has been devoted to development of new, more computationally economic probabilistic algorithms for advanced reliability and importance sampling methods in the GENPAM program as a direct result of ten years of probabilistic structural analysis research funded by NASA.

The GENPAM code is constructed such that any real value in the input file of the fracture toughness, and fatigue crack growth deterministic analysis can be selected as a random variable (Table 1). An interface module was developed that can interface with any deterministic code as long as the uncertainties are one of the real values shown on the original deterministic input file. Integration with many commercial or in-house computer codes becomes transparent. Thus, integration effort is minimized and simplified. Various responses can be selected to be analyzed probabilistically, CDF/PDF functions and sensitivities to design random variables. The types of responses that can be specified are: 1) Type 1: fracture toughness, and 2) Fatigue Crack Growth responses for the accelerated, paris, and threshold regions.

The probabilistic analysis code takes into account the uncertainties in material properties. Uncertainties in all the relevant design variables are quantified for determination of their effects on fracture toughness, and progression. A probabilistic analysis cycle starts with defining uncertainties in material properties at the most fundamental constituents.

Consequently, Probability Density Functions and cumulative distribution functions (CDF) can be obtained. Sensitivities of various design variables to material response are also obtained. Input data for probabilistic analysis is generated from the design variables

with probabilistically defined uncertainties and the response parameters that are to be analyzed probabilistically are user selectable.

Table 1. Material Uncertainties Considered

Fracture Toughness	Fatigue Crack Growth
Yield strength	Kc Critical Stress Intensity Factor(Accelerated region)
Ultimate Strength	Kth Thersshold Stress Intensity Factor
Rupture Strength	$\alpha_1, \alpha_2, \beta_1, \beta_2$ in Paris region (see section 2.2)

3.0 Results and Discussions

3.1 Probabilistic evaluation of fatigue crack growth analysis - considered 5 to 10% coefficient of variation of 2219-T87 aluminum material and Kc, and Kth as random variables. The probabilistic evaluation of the above selected alloy determined : 1) shift in both fracture toughness versus material thickness and fatigue crack growth (da/dn versus ΔK) plots (Figure 11) for the above alloy, 2) sensitivity of the random variables (Kc, Kth) to response variables (Figure 12), 3) Probability Density Function of fatigue crack growth properties in the Paris region (Figure 13) and 4) cumulative distribution function demonstrating the probability of crack growth rate (Figure 14).

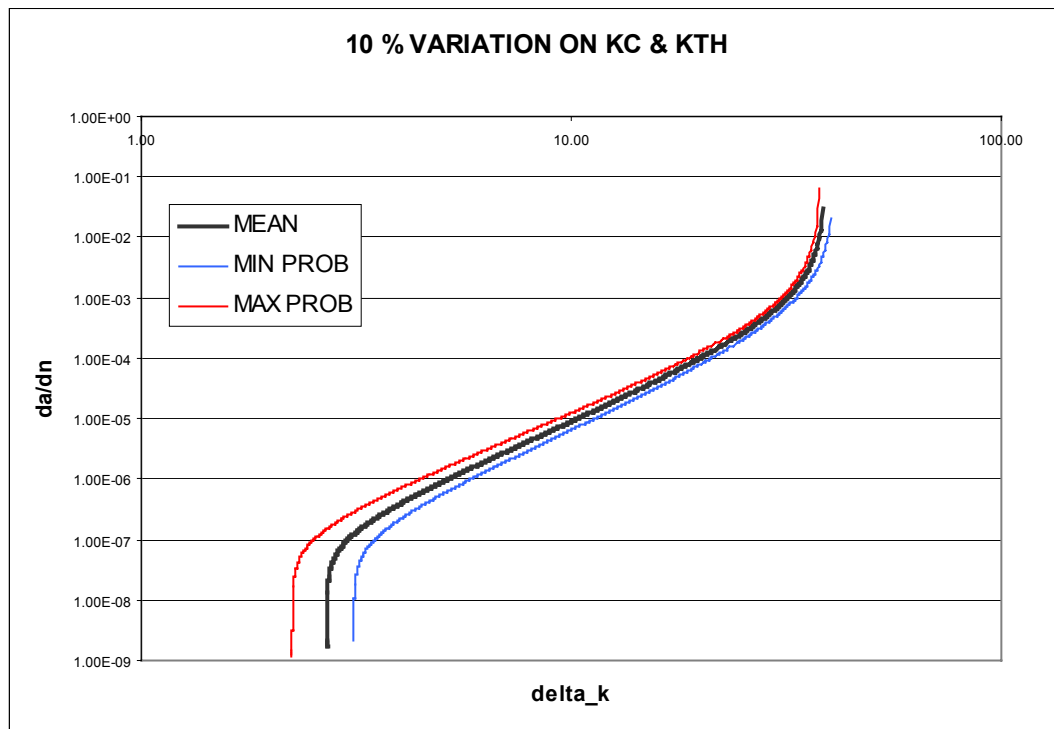


Figure 11: Sensitivity Run On Fatigue Crack Growth Properties As A Result Of Material Variation

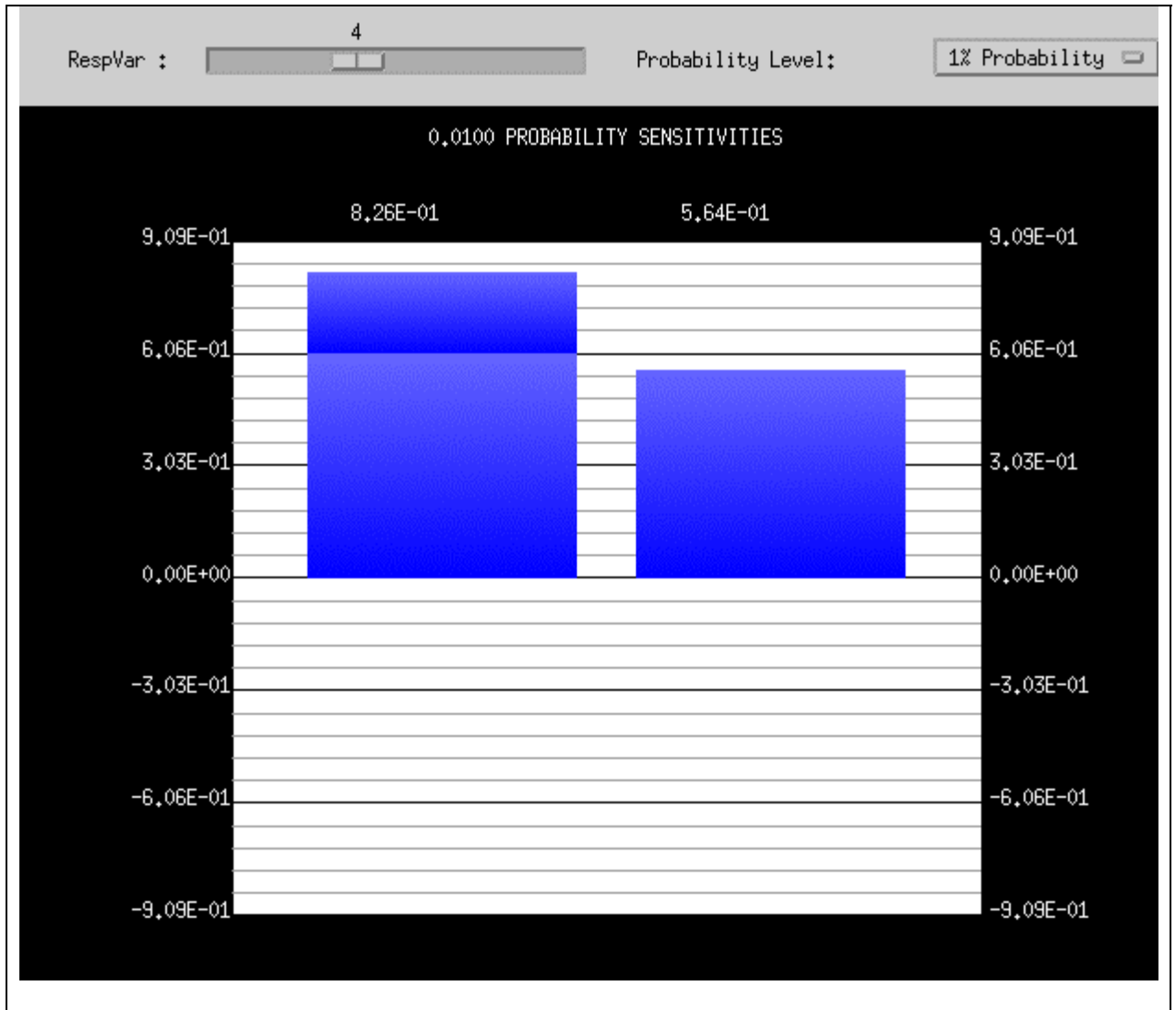


Figure 12: Sensitivity of Material (KC, KTH) on Fatigue Crack Growth Properties in Paris region

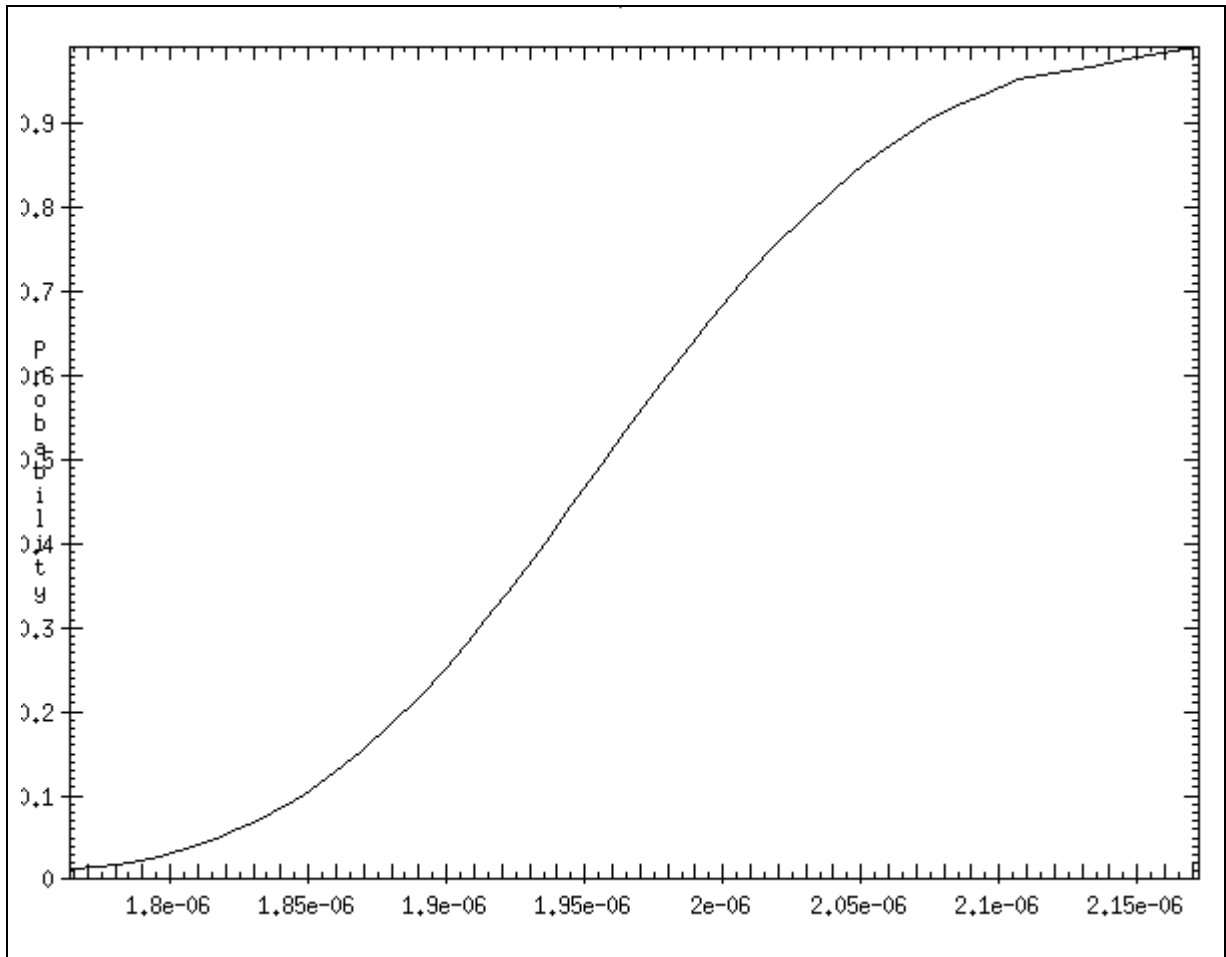


Figure 14: Cumulative Distribution Function of Fatigue Crack Growth Properties in the Paris Region

3.2 Probabilistic Evaluation of Fracture Toughness - Normal distribution and 5% coefficient of variation of random variables (ultimate strength, yield strength, and necking strength) for the aluminum 2219-T87 material is considered. The probabilistic evaluation of the above selected alloys determined the shift in fracture toughness versus material thickness plots. Figure 15 shows the fracture toughness variation versus thickness of the above selected alloy. Figure 16 shows the sensitivity of the random variables: 1) ultimate strength, 2) yield strength, and 3) necking strength for the aluminum 2219-T87 material to fracture toughness response variables. Figure 17 shows the Probability Density Function of the fracture toughness. Figure 18 shows the probability of fracture toughness by the cumulative distribution function.

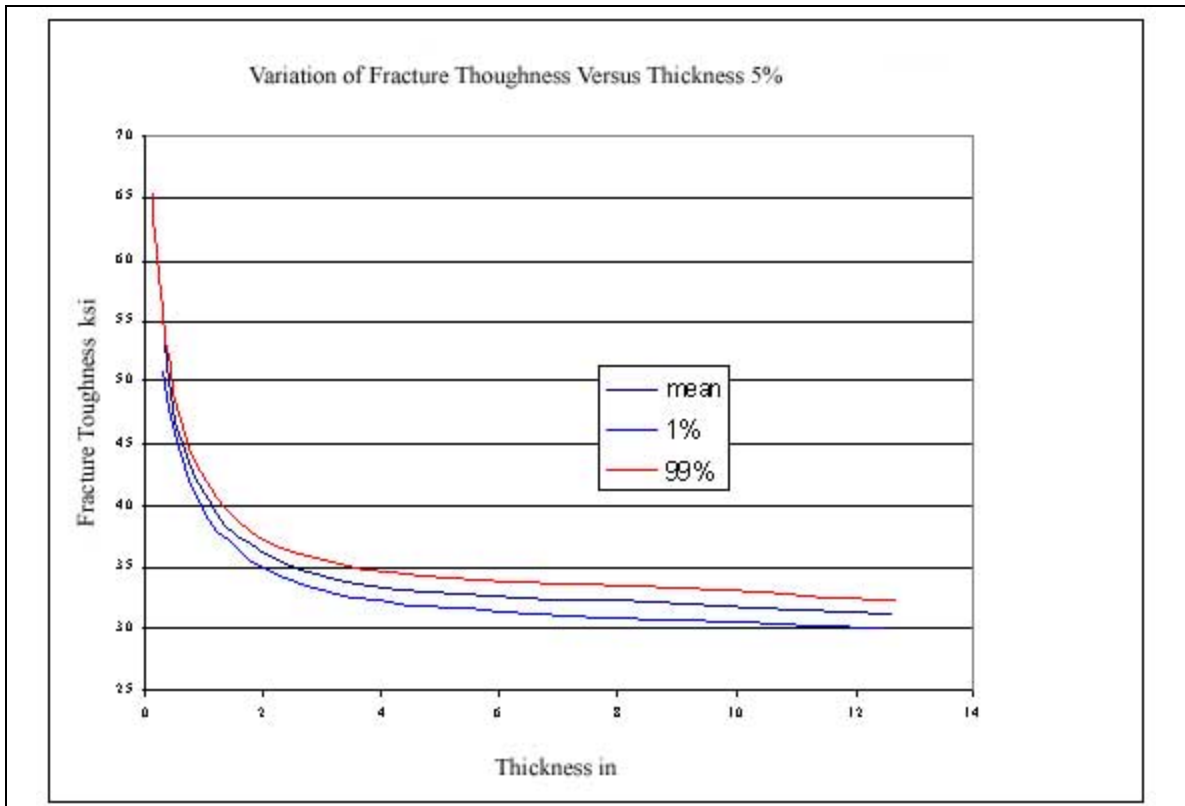


Figure 15: Sensitivity Run on Fracture Toughness Properties as a Result of Material Variation

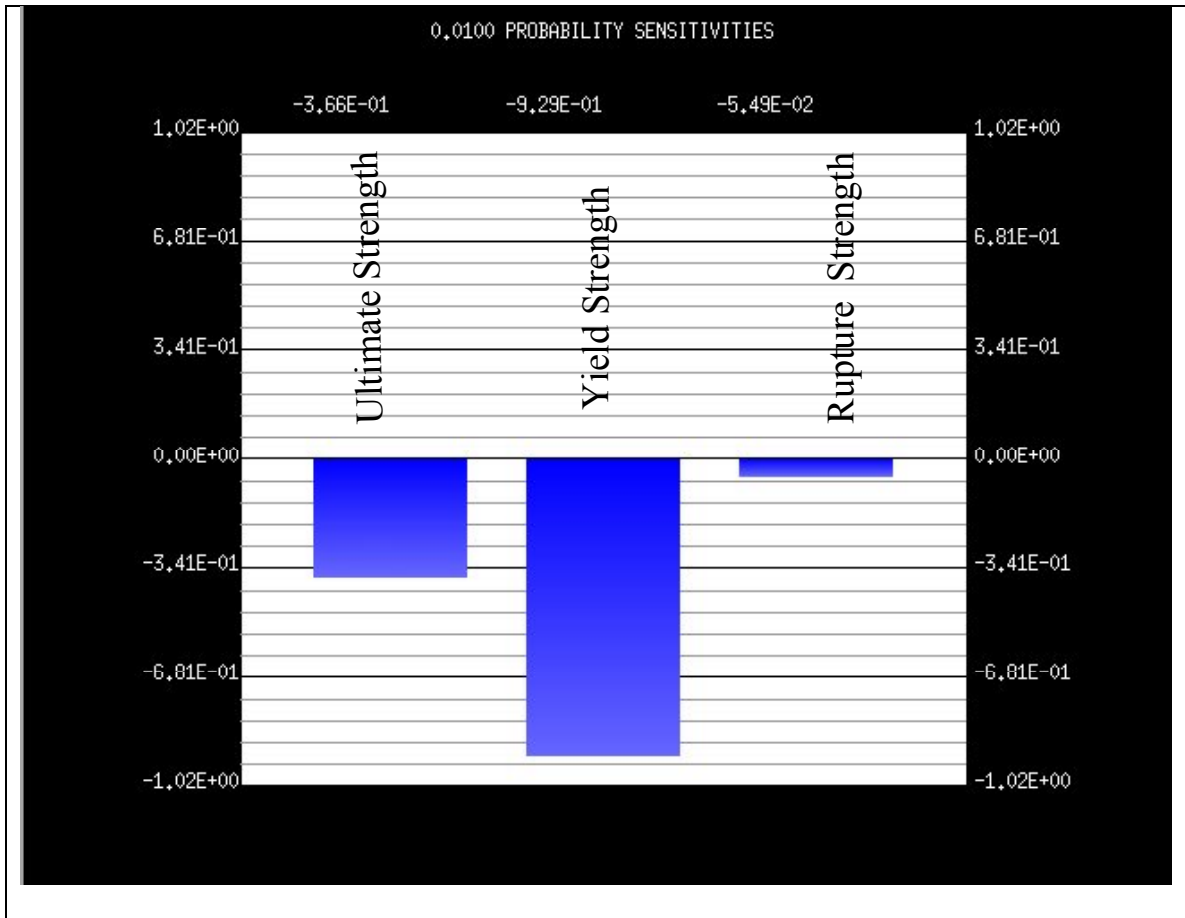


Figure 16: Sensitivity of Material Strength(Ultimate, Yield, and Neck) on Fracture Toughness

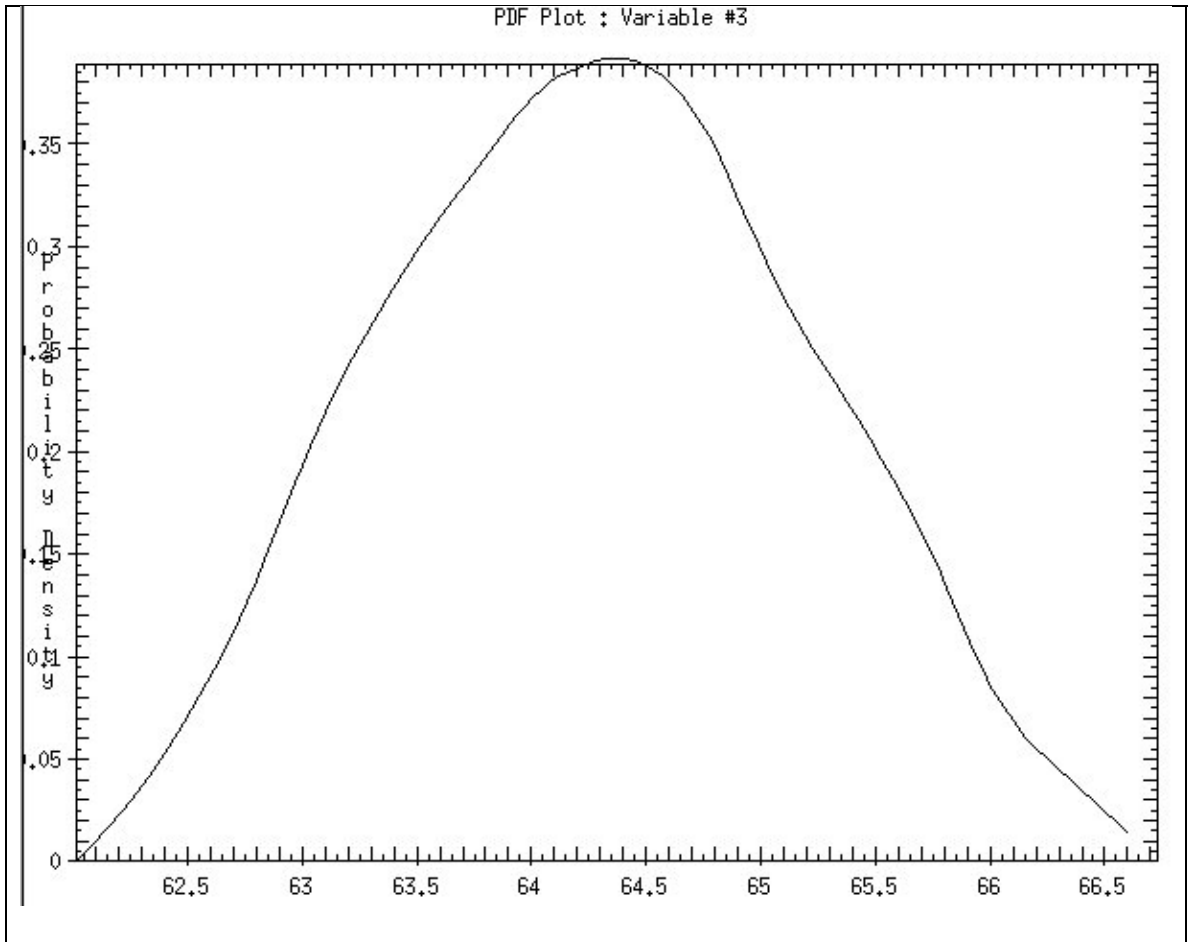


Figure 17: Probability Density Function of Fracture Toughness

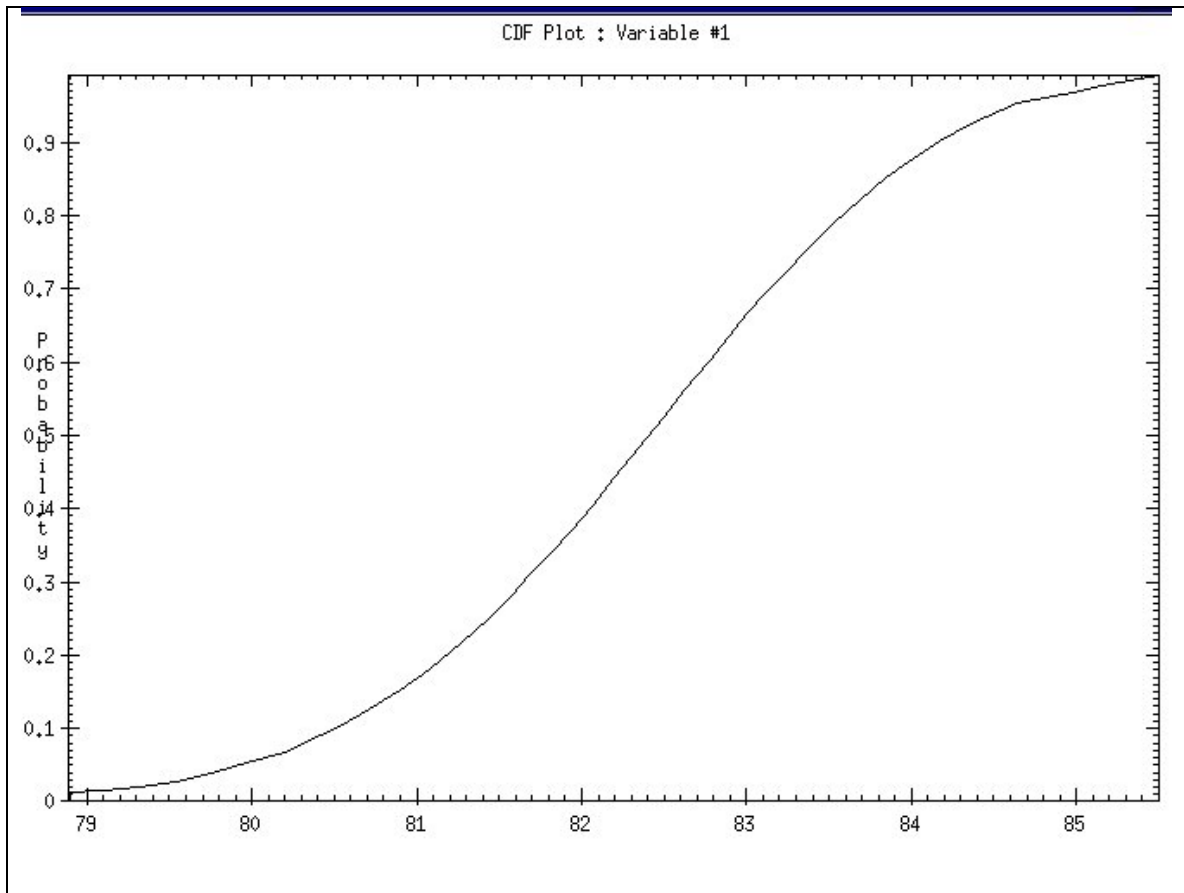


Figure 18: Cumulative Distribution Function of Fracture Toughness

4.0 Conclusion

The proposed approach can generate fracture toughness, and fatigue crack growth data for classical metal alloys used in the Aerospace industry. In addition the probabilistic methods integration has allowed a better understanding of the scatter effect on material reliability, and life prediction by:

- 1 Material plane strain and stress fracture toughness are sensitive to material variations observed in the full stress-strain curve.
- 2 Material fatigue crack growth curve is sensitive to parameters that contribute to the Threshold , Paris, and Accelerated regions.
- 3 Probabilistic study has shown that both fracture toughness versus material thickness and fatigue crack growth curves will shift depending on material variations observed through static tests.

5.0 References

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- [2] B. Farahmand, "Fracture Mechanics of Metals, Composites, Welds, and Bolted Joints," Kluwer Academic Publisher, Nov. 2000, Chapter 5.
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- [4] Fatigue Crack Growth Computer Program "NASA/FLAGRO", Developed by R. G. Forman, V. Shivakumar, J. C. Newman. JSC-22267A, January 1993.
- [5] B. Farahmand, "Virtual Testing Approach for Determination of Fatigue Crack Growth and Material Fracture Toughness," Boeing Technical Excellent Conference, Seal Beach, California, Feb., 2000.